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GROUP I. PART II.—*Continued.*

$$J =$$

$$25.* \frac{1}{3} \frac{m_a^2 + m_b^2 + m_c^2}{\cot A + \cot B + \cot C}$$

$$26. \sqrt{2R\beta_a\beta_b\beta_c [1 + \cos(A - B) + \cos(B - C) + \cos(C - A)]}$$

$$= 2\sqrt{R\beta_a\beta_b\beta_c \sin(A + \frac{1}{2}B) \sin(B + \frac{1}{2}C) \sin(C + \frac{1}{2}A)}$$

$$27. R\sqrt{l_a l_b l_c} \sin A \sin B \sin C$$

$$28. \frac{1}{3}R \sin A \sin B \sin C \left( \frac{ab}{l_c} + \frac{bc}{l_a} + \frac{ca}{l_b} \right)$$

$$29. \frac{l_a + l_b + l_c}{2 \left( \frac{\cos \frac{1}{2}A}{\beta_a} + \frac{\cos \frac{1}{2}B}{\beta_b} + \frac{\cos \frac{1}{2}C}{\beta_c} \right)}$$

$$30. \frac{\beta_a \sin(C + \frac{1}{2}A) + \beta_b \sin(A + \frac{1}{2}B) + \beta_c \sin(B + \frac{1}{2}C)}{2 \left( \frac{\cos \frac{1}{2}A}{\beta_a} + \frac{\cos \frac{1}{2}B}{\beta_b} + \frac{\cos \frac{1}{2}C}{\beta_c} \right)}$$

$$31. \frac{\beta_a \beta_b \beta_c \sqrt{(m_a^2 - l_a^2)(m_b^2 - l_b^2)(m_c^2 - l_c^2)}}{s(a - b)(b - c)(c - a)}.$$

[TO BE CONTINUED].

## DEMONSTRATION OF DESCARTES'S THEOREM AND EULER'S THEOREM.

By PROF. G. B. HALSTED, Austin, Texas.

## DESCARTES'S THEOREM.

Cutting by diagonals the faces not triangles into triangles, the whole surface of any polyhedron contains a number of triangular faces four less than double the number of summits.

*Proof.*

For, joining all the summits by a single closed broken line, this cuts the surface into two skew polygons, each of which contains  $S - 2$  triangles, where  $S$  is the number of summits.

## EULER'S THEOREM.

The number of faces and summits in any polyhedron taken together exceeds by two the number of its edges.

*Proof.*

First Case. If all the faces are triangles; then by Descartes's theorem,

$$F = 2(S - 2).$$

But also  $2E = 3F$ , since each edge belongs to two faces, and so we get a triangle for every time 3 is contained in  $2E$ . By adding, we have

$$2E = 2F + 2(S - 2),$$

that is,

$$F + S = E + 2.$$

Second Case. If not, all the faces are triangles. Since to any summit go as many faces as edges, we may replace any polygonal face by a pyramidal summit without changing the equality or inequality relation of  $F + S$  to  $E + 2$ ; for such replacement only adds the same number to  $F$  as to  $E$  and changes one face to a summit. But after all polygonal faces have been so replaced,  $F + S = E + 2$  by our first case. Therefore the relation is always equality.

[These theorems are substantially one; the second is also due to Descartes, having been published in 1860 in his *Œuvres Inédites*.—*W. M. T.*]

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 PROOF OF A PROPOSITION IN MODERN GEOMETRY.

By MR. R. D. BOHANNAN, University of Virginia.

$A, B$  are any two points on the curve which is the locus of the intersection of corresponding rays of two homographic ray-systems. If  $A, B$  be made the centres of two ray-systems whose corresponding rays intersect on this curve, these two ray-systems are homographic.

The curve which is the locus of corresponding rays of two homographic ray-systems is of the second degree and passes through the two ray-centres. Being of the second degree, it is fixed by fixing on it five points  $A, B, C, D, E$ . Take on it any sixth point  $F$ . The three rays  $BC, BD, BE$  may be taken arbitrarily to correspond to the three rays  $AC, AD, AE$ . Suppose the ray  $BK$  corresponding to  $BF$  does not intersect  $AF$  in  $F$ , but in  $K$ . Then we have two curves of the second degree, one passing through the six points  $A, B, C, D, E, F$  and the other passing through the six points  $A, B, C, D, E, K$ . But the curves have five points in common. Thus  $K, F$  are coincident. Therefore, etc.